Adaptive high order Finite Element modelling of poroelastic materials

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This paper presents a physics-based adaptivity algorithm to determine the directional element orders for poroelastic material models built-up through the ($\mathbf{u}^s$,p^f)-formulation of the Biot equations. The methodology is applied to a use case to determine the plane wave absorption coefficient of a poroelastic meta-material with an elastic inclusion.

1 Introduction

With ever more stringent requirements on noise and vibration levels, engineers strongly rely on passive treatments (e.g. multi-layered stacks of of elastic, viscoelastic and poroelastic layers) to improve the sound and vibration properties of a product. Over the past decades, CAE tools built upon the Finite Element Method (FEM) have become indispensable tools to predict the behaviour of virtual prototypes in an early design stage. Through its innate element discretisation concept, the FEM can tackle geometrically complex problem cases. Within these elements, polynomial approximation functions are used to describe the dynamic field variables. This procedure is a strength of the FEM, but it constitutes also a limitation. As the frequency increases, the number of low-order elements required to control the interpolation and pollution errors increases more than linearly. In order to balance accuracy and computational cost, frequency-dependent input meshes are often required. On top of this, poroelastic materials pose additional constraints as they have a high number of DOFs per node and support relatively short wavelengths and outspoken near-field damping effects.
To overcome these problems, research been done over the past years to improve the efficiency of FE methods. Initial steps into p-refinement (order refinement) were made by Hörlin [3] and Rigobert [6]. However, due to the innate coupling between both phases, together with the highly dispersive properties of the different wave types that propagate in a poroelastic material, it still is still highly challenging to derive simple guidelines to create a valid mesh (and FE model). Hereto, this paper proposes a higher order, automated adaptive FE procedure for poroelastic material modeling. The element orders are determined *a priori*, adapted to the frequency and based on the physics of poroelastic materials. Furthermore, these element orders can be assigned independently in different directions, in order to account for anisotropy in the mesh. The result is a (nearly) mesh-independent solution that allows to reach a user-defined target error while minimizing the computational cost.

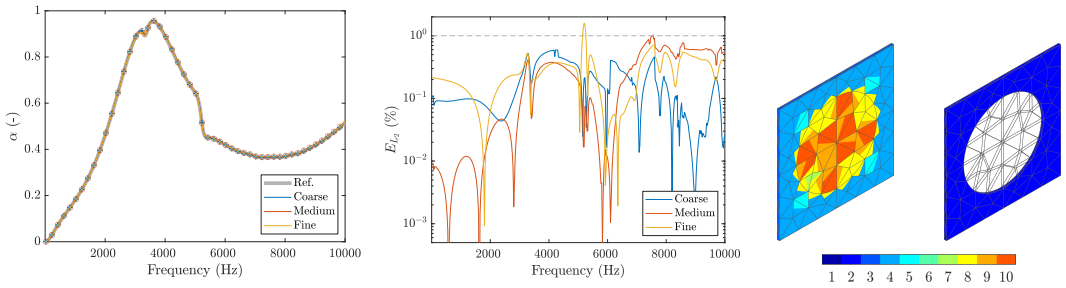
2 Adaptive high order finite element model

The variational formulation of the Biot equations is discretised using the high-order finite element method [2, 4]. On each element, the numerical solution is described using Lobatto shape functions. These form a 'hierarchic' basis. It is therefore simple to handle models with different polynomial orders in the element. Furthermore, the hierarchic p-FEM scheme allows to enforce directional orders in the interpolation basis.
The directional element orders are built up bottom-up from the edge orders using specific element conformity rules [1] to determine the 2D- and 3D elements' directional orders. The element edge orders are determined by comparing the accuracy at given order with the solution of a reference problem, describing the propagation at oblique incidence at an angle θ, which is the most general possible problem on a 1D element topology. The dependence in the element direction to the parameter θ is low, so a value of 30° is used. The element edge orders p_{solid} and p_{fluid} are chosen to minimise the computational cost, subject to an error treshold E_T. Since the computational cost is directly influenced by the element order, the optimisation problem can be translated to minimising the weighted sum of solid- and fluid phase orders. However, the solution of this optimisation problem is not trivial. Nevertheless, in a first step the monotonous convergence of the FEM with increasing order can be

exploited. Therefore p_{solid} and p_{fluid} are increased simultaneously until $E_{L^2} < E_T$. This accords with the method of steepest descent. Thereafter, either p_{solid} or p_{fluid} can be reduced by tracing the error threshold isoline $E_{L^2} = E_T$ horizontally or vertically. p_{solid} is reduced first (if possible) as it has the highest influence on the computational cost. This strategy ensures that the set E_T is met and the computational cost is minimised.

3 Use Case: Poroelastic meta-material with elastic inclusion

The acoustic plane wave absorption in the normal direction of a poroelastic metamaterial with an elastic inclusion [7] is studied as an application. Figure 1(a) shows the predicted acoustic absorption curves for three different meshes. For all, the results are in excellent agreement with the reference model and globally satisfy the set tolerance $E_T = 1\%$ as the error curves in Figure 1(b) indicate. Figure 1(c) shows a deeper insight in the adaptive order structure for the model at 10 kHz, showing the minimal and maximal orders for both the displacement and pressure variables.



(a) Absorption coefficient (b) L^2-error w.r.t reference (c) $\mathbf{u}$ and p orders

Figure 1: Results for the absorption analysis of a poroelastic meta-material with inclusion.

4 Conclusion

This paper focuses on an efficient implementation of a higher-order Finite Element Method for poroelastic materials described by the Biot theory. Crucial is a criterion based on a 1.5D reference problem to determine the solid- and fluid phase orders semi-independently, minimising the computational cost for a required accuracy. The calculated edge orders can are then propagated to the face (2D) and solid (3D) directional element orders by using some element-type specific conformity rules. The approach is applied with success to the absorption analysis of a poroelastic meta-material with inclusion.

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