# A Poro-Elastic Model for Activated Carbon Stacks

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# 1 Introduction

Porous materials are widely applied in the field of noise control owing to their ability to attenuate transmission and/or reflection of sound. Among these materials, activated carbon is of increasing interest due to its excellent absorption performance at low frequencies. In 2016, a rigid, triple porosity model for granular activated carbon (GAC) was proposed by Venegas and Umnova [1]. In the proposed model, a sorption process was introduced to account for the effect of the smallest scale pores, which is correlated with the high absorption at low frequencies [1,2]. To extend that model to account for the finite frame stiffness of the particle stack, a poro-elastic model for a stack of granular activated carbon is introduced here. The new model combines aspects of the Biot theory [4,5,6] and the rigid GAC model [1,2,3]. The stable computational approach proposed by Dazel et al. [7] was followed when implementing the poro-elastic model. A fitting procedure based on particle swarm optimization (PSO) was applied to identify the parameters of proposed model that then yield predictions closely matching measurement of the absorption coefficient of rigid-backed activated carbon stacks.

# 2 Proposed poro-elastic model

The GAC model is based on the assumption that the particles are spherical, and that the pores within the particles are cylindrical [1,2]. In the proposed model, the volume change of the inter-particle interstices caused by the frame elasticity is considered. Therefore, the fluid phase bulk modulus and equivalent density were obtained from the GAC model, while the macro porosity, $\phi_p$, in the GAC model, was treated as the porosity of the whole stack.

# 3 Parameter fitting

The parameters of the proposed model were obtained by fitting the model prediction to measured absorption coefficients. The fitting procedure was implemented *via* the PSO algorithm, which was realized by using the toolbox 'Constrained Particle Swarm Optimization' [8]. Since the bulk density for the activated carbon particles can be measured, that information was utilized as a nonlinear constraint applied to the porosities. In the cases shown in the next section, the bulk density inferred from fitted porosities was constrained to lie within $\pm 5\%$ of the measured value. Two arrays of weightings applied to different frequencies were defined for the measurements obtained using a small tube (diameter 2.9 cm) and medium tube (diameter 6.35 cm), respectively. The weightings at very low frequencies were set to be zero to discard the unreliable measurement in that range. The weightings around the frequency where a quarter-wave frame resonance peak appears were set to 2, while the weighting at all other frequencies was set to 1. The intention here was to help ensure that the fitting procedure yielded results that accurately captured the frame resonance feature Therefore, the optimization problem can be expressed as,

$$\begin{aligned} \min_{\mathbf{x}} \quad & f(\boldsymbol{x}) = \mathbf{w}^{\mathrm{T}}(\mathbf{a} - \mathbf{a}_m)^2/N \\ \text{s.t.} \quad & \mathbf{x}_{lb} < \mathbf{x} < \mathbf{x}_{ub} \\ & 0.95\rho_m < \rho_c(1-\phi_{tb}) < 1.05\rho_m \end{aligned} \qquad [1]$$

where $N$ denotes the number of measured data points; $\mathbf{x}$ represents the parameter vector of the poro-elastic model, which consists of frame stiffness, Poisson's ratio, loss factor, macroscopic porosity, mesoscopic porosity, microscopic porosity, particle radius, mesopore radius, micropore radius, Langmuir constant, and configurational diffusivity, with $\mathbf{x} = (E,\ \nu, \eta, \phi_p, \phi_m, \phi_n, r_p, r_m, r_n, b, D_c)$; $\mathbf{x}_{lb}$ and $\mathbf{x}_{ub}$ denote the lower and upper bound of the parameters; $\mathbf{a}$ and $\mathbf{a}_m$ denote predicted and measured absorption coefficient, respectively; and $\mathbf{w}$ denotes the weighting vector applied on the measurements.

# 4 Fitting results

The fitting results for 30mm-thick stacks of two types of activated carbon are shown in the figure below.

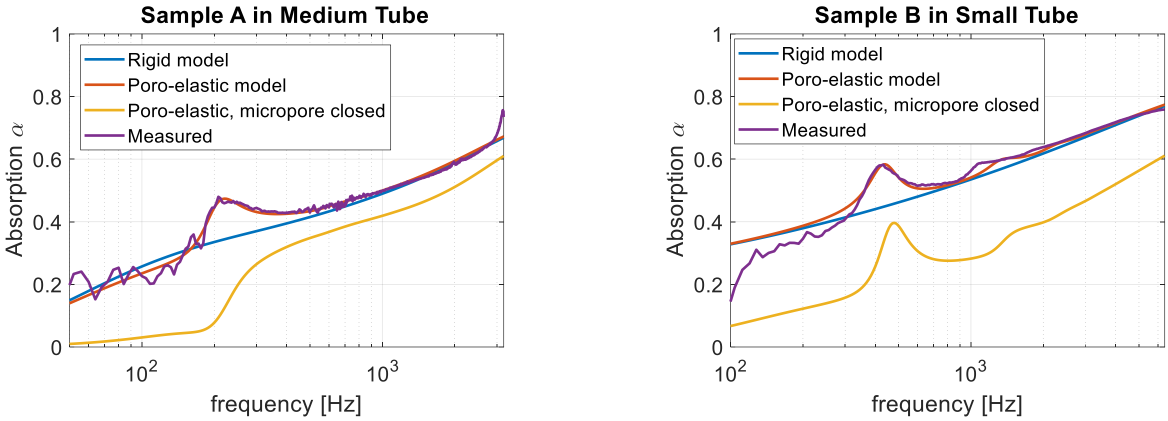


Figure 1: The absorption coefficient from fitted parameters compared with measurements.

In both cases, the fitted parameters capture the resonance peak at the frequency where the stack thickness equals one-quarter of the longitudinal structural wavelength. The predictions of the rigid model do not show that peak. In the second case, the fitted parameters can also capture the small second peak at the frequency where the stack thickness equals three-quarters of the longitudinal structural wavelength. Further, when the micropores are “turned off”, the absorption is reduced, as shown by the yellow line, thus demonstrating the low frequency impact of the sorption process.

# 5 Conclusion

The proposed poro-elastic model can predict the high frequency asymptotic behavior of the activated carbon stack as well as the rigid model, but in addition, it can also reproduce the frame resonance peak(s) in the absorption coefficients, with physically reasonable parameters. When comparing the absorption coefficient with micropores open and closed, it can be seen that the existence of micropores boosts the absorption significantly.

# References


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